# Time-resolved and time-averaged determination of the electron-ion collision frequency and electron stopping force in an intense laser field at critical plasma density

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Using a ballistic dynamic equation (oscillator model) in the presence of an intense laser field ($v_{\rm osc} \gg v_{\rm th}$) the time-resolved and time-averaged electron stopping force and electron-ion collision frequency are determined at the critical density ($\omega_0 = \omega_p$) in the presence of laser field harmonics. An expression for the time-averaged energy absorption rate is given. Results show the contributions of single-particle effects and the generated laser harmonics to the electron-ion collision frequency and energy absorption rate. This work also discusses the time-resolved electron-ion collision frequency.

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## I. INTRODUCTION

The interaction of intense laser fields with matter has become a topic of great interest in theoretical and experimental research due to the formation of high-density nonideal plasmas. The understanding of such interaction mechanisms and related phenomena as well as their mathematical modeling are of importance for energy transport from the outer coronal plasma region into deeper plasma layers [1–5]. Transport processes of laser field deposited energy and corresponding transport coefficients are strongly dependent on the energy absorption rate taking place in the plasma corona.

Electrical conductivity is one of the important quantities for the understanding of interaction and coupling mechanisms between laser radiations and matter. It represents a measure for interaction and coupling strengths. Electron-ion collisions are in turn of importance for calculating the electric conductivity in fully and partially ionized gases, as well as for plasma heating and damping of waves that might be excited or generated in plasmas by the coupling between natural plasma modes and the incident laser field [6–9].

Wave damping is important for plasma heating and can be treated as a combination of collisional and collisionless mechanisms. Epperlein *et al.* showed that weak electron-ion collisions increase the damping rate above the collisionless electron Landau damping [7]. In addition, taking into account particle trapping in the wave potential well and electron collisions with neutrals, Kaganovich showed that the difference between the results obtained within the linear theory on collisionless power dissipation can be as large as three orders of magnitude [8]. Procassini and Birdsall investigated particles and energy transport processes in fully ionized collisional plasma using a kinetic transport model [9]. They presented the variation of several transport quantities with plasma collisionality and found a reduction in the heat conduction flux compared with the values predicted by the classical hydrodynamic transport theory [10].

Many works have considered collisions of charged particles in the presence of a magnetic field [11–13]. Imazu investigated the collision frequency of charged particles in a weakly ionized gas in a strong magnetic field. He found that the collision frequency of electrons or ions in the presence of the magnetic field is smaller than that when the magnetic field is absent [11]. Panov investigated the effect of dynamical screening on photon absorption in screened Coulomb interactions of electrons with ions in degenerate plasmas in the presence of a quantized magnetic field [12]. He investigated the dependence of the effective collision frequencies in the directions along ($\nu_\parallel$) and perpendicular ($\nu_\perp$) to the external magnetic field on photon frequencies $\omega_0$ less than the cyclotron frequency $\omega_c$. Panov's results show that a peak value of the collision frequency occurs at $\omega_0 = 0.5\omega_c$.

Absorption of rf power by a plasma in the presence of a perpendicular alternating electric field and a constant magnetic field has been investigated experimentally by Naumovets *et al.* [13]. An effective collision frequency was calculated from the experimental data and was found to be an order of magnitude larger than the kinetic collision frequency. In crossed dc electric and magnetic fields, Nakamura *et al.* studied the scaling of electron swarm parameters in argon and methane using Monte Carlo computational experiment [14]. For very high magnetic field, they found that the electron mean energy and collision frequency become insensitive to magnetic field variations.

Bohm and Pines have classified Coulomb collisions into two component. One component represents collective oscillations and is due to long-range interactions at distances greater than the Debye length $\lambda_D$, and the other component is due to individual particle interactions within a Debye sphere of the radius $\lambda_D$. For plasma particles possessing only random thermal motions or moving in low-intensity (weak) fields, the maximum extent of long-range interactions can be approximated by the Debye length $\lambda_D$. This is due to the plasma tendency toward shielding electric fields on a scale length greater than $\lambda_D$, and therefore long-range interactions will be screened out by a shielding cloud near the scattering center.

Coulomb forces between charged particles have a much longer range than the forces between charged and neutral particles. As a result, collisions of charged particles cannot be considered as events occurring when the particles are

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close to each other. Because of the long-range ($1/r$) Coulomb potential, a charged particle is nearly always interacting with many particles simultaneously. In plasmas, for example, electrons collide with each other, with ions, and with neutral particles. The collision process consists of two point charges moving on hyperbolic paths in each other's electric field.

Rutherford's theory of binary charged-particle collisions ignores polarization effects of the medium. As a result, particles interact through an unscreened Coulomb field. At large distances or small angle deflections (forward scattering) corresponding to soft interactions, a divergence will appear. Energy and momentum exchanges for Coulomb deflections from 0 to some distance $b$ vary as $\ln(b/b_\perp)$, where $b_\perp$ is the $\pi/2$ impact parameter. Due to the use of the exact colliding particle trajectory, large-angle deflections or backward scattering corresponding to near collisions lead to a convergence behavior for small distances. Removal of the divergence at large distances is achieved usually by introducing a cutoff parameter $b_{\max}$.

Another way to overcome singularities at large distances is to use a screened Coulomb potential or the Debye Hückel potential $[\phi=(q/4\pi\epsilon_0 r)e^{-r/\lambda_D}]$ [15]. This leads to $b_{\max}=\lambda_D$ and is valid for plasmas in which thermal motion is dominant or for low-intensity fields. For plasmas in high frequency fields, the corresponding electrical conductivity and collision frequency were investigated classically and quantum mechanically in many works. Most of the results are in a good agreement with each other [16–19].

Cornolti *et al.* investigated the absorption of ultrashort laser pulses in solid targets using a ballistic model [20]. It has been found that electron-ion collision frequency at high laser intensities ($\sim 10^{17}\,\mathrm{W/cm^2}$) is determined by the oscillation energy of the electrons rather than by their thermal motion. For high-intensity irradiation corresponding to electron quiver velocity $v_{\mathrm{osc}}$ much greater than the random thermal speed $v_{\mathrm{th}}$, this behavior of the electron-ion collision frequency was predicted by many previous works, but there were no quantitative unified results [19–24].

In this work we use a ballistic model, known in the literature as the oscillator model [20,25], to investigate electron stopping force and electron-ion collision frequency in strong laser fields. In Sec. II we present the oscillator model and solve the corresponding dynamical equation. In Secs. III and IV we determine the time-resolved and time-averaged stopping force and collision frequency. In Sec. V we present our results and conclusions.

## II. OSCILLATOR MODEL

Linearizing the following equations of the electron fluid,

$$\frac{\partial n_e}{\partial t}+\vec{\nabla}\cdot(n_e\vec{u}_e)=0, \tag{1}$$

$$\frac{\partial \vec{u}_e}{\partial t}+(\vec{u}_e\cdot\vec{\nabla})\vec{u}_e=-\frac{e}{m_e}(\vec{E}+\vec{u}_e\times\vec{B}), \tag{2}$$

$$\vec{\nabla}\cdot\vec{E}=\frac{\rho_{0i}+\rho_e}{\epsilon_0}, \tag{3}$$

Mulser *et al.* [25] derived the following equation for the calculation of collisional absorption in cold plasmas:

$$\frac{\partial^2\vec{\delta}}{\partial t^2}+\omega_{\mathrm{pe}}^2\vec{\delta}=-\frac{Ze^2}{4\pi\epsilon_0 m_e}\frac{\vec{r}_e(t)}{|\vec{r}_e(t)|^3}, \tag{4}$$

where $\vec{\delta}$ is the electron displacement or deviation from the equilibrium position, $\omega_{\mathrm{pe}}$ is the electron plasma frequency, and $|\vec{r}_e(t)|$ is the relative separation between the singled out electron-ion pair. Positive ions were considered as an immobile neutralizing background with uniform density $Zn_{0i}=n_{0e}$, where $n_{0e}$ is the equilibrium density of electrons.

Ashley *et al.* used a classical equation similar to Eq. (4) to investigate the $Z^3$ effect in the stopping power of matter for heavy charged particles of charge $Ze$ [26]. Single electrons were considered to be bound isotropically and harmonically with frequency $\omega$ to the nucleus of the atom. Contrary to the Ashley *et al.* approach, the right-hand side of Eq. (4) represents, in the ion frame, the force set up by an oscillating ion on a stationary electron bound harmonically to the plasma with a frequency $\omega_p$. Relative electron-ion separation can be approximated as follows:

$$\begin{aligned}\vec{v}_e(t)&=\vec{v}_{\mathrm{osc}}+\vec{V}_0=(V_{0x}+v_{\mathrm{osc}}\sin\omega_0 t)\hat{i}+V_{0y}\hat{j},\\ \vec{r}_e(t)&=(V_{0x}t\pm y_{\mathrm{osc}}\cos\omega_0 t)\hat{i}+(V_{0y}t+b)\hat{j},\end{aligned} \tag{5}$$

where $\vec{V}_0, b, \omega_0, v_{\mathrm{osc}}$, and $y_{\mathrm{osc}}$ are the average drift velocity of the electron fluid, the collision parameter, the laser frequency, the electron quiver velocity, and the electron oscillation amplitude in the laser field, respectively. In Eq. (5) the incident laser field is taken along the $x$ axis. Upon substituting $\vec{r}_e(t)$ from Eq. (5) and assuming that the electron drift $(V_0 t)$ is small compared with $y_{\mathrm{osc}}$, Eq. (4) can be written to a first approximation as follows:

$$\frac{\partial^2\vec{\delta}}{\partial t^2}+\omega_{\mathrm{pe}}^2\vec{\delta}\approx\frac{\alpha\vec{r}_e(t)}{[V_0^2t^2+b^2+y_{\mathrm{osc}}^2\cos^2\omega_0 t]^{3/2}}, \tag{6}$$

where

$$\alpha=-\frac{Ze^2}{4\pi\epsilon_0 m_e}. \tag{7}$$

Equation (6) describes in fact the interaction of three bodies: electrons, ions, and photons of the incident laser radiation. The right-hand side of Eq. (6) shows through the dependence on the instantaneous laser field strength that ions are used as an intermediary for energy transfer between electrons and the laser field. The electron deflection $\vec{\delta}$ has therefore a nonlinear dependence on the electric field of the laser, and this will lead to the possibility of the occurrence of radiation harmonics.

The general solution for Eq. (6) is

$$\begin{aligned}\delta_x(t)=A_x\cos\omega_p t+B_x\sin\omega_p t-\alpha\frac{\cos\omega_p t}{\omega_p}I_1(t)\\ +\alpha\frac{\sin\omega_p t}{\omega_p}I_2(t),\end{aligned} \tag{8}$$

$$\delta_y(t)=A_y\cos\omega_p t+B_y\sin\omega_p t-\alpha\frac{\cos\omega_p t}{\omega_p}I_3(t)+\alpha\frac{\sin\omega_p t}{\omega_p}I_4(t), \tag{9}$$

where

$$I_1(t)=\int^t dt\frac{y_{\rm osc}\sin\omega_p t\cos\omega_0 t}{(b^2+y_{\rm osc}^2)^{3/2}[\sqrt{1-K^2(t)\sin^2\omega_0 t}]^3}, \tag{10}$$

$$I_2(t)=\int^t dt\frac{y_{\rm osc}\cos\omega_p t\cos\omega_0 t}{(b^2+y_{\rm osc}^2)^{3/2}[\sqrt{1-K^2(t)\sin^2\omega_0 t}]^3}, \tag{11}$$

$$I_3(t)=\int^t dt\frac{b\sin\omega_p t}{(b^2+y_{\rm osc}^2)^{3/2}[\sqrt{1-K^2(t)\sin^2\omega_0 t}]^3}, \tag{12}$$

$$I_4(t)=\int^t dt\frac{b\cos\omega_p t}{(b^2+y_{\rm osc}^2)^{3/2}[\sqrt{1-K^2(t)\sin^2\omega_0 t}]^3}, \tag{13}$$

$$K^2(t)=\frac{y_{\rm osc}^2}{V_0^2t^2+b^2+y_{\rm osc}^2}, \tag{14}$$

$$\omega_{\rm pe}^2=\frac{n_e e^2}{\epsilon_0 m_e},\quad y_{\rm osc}=\frac{eE_0}{m_e\omega_0^2}. \tag{15}$$

The assumption $y_{\rm osc}\gg V_0 t$ is valid for small electron drift motion. Upon evaluation of the undetermined integrals $I_1(t),I_2(t),I_3(t)$, and $I_4(t)$ in this limit, the solution of Eq. (8) which satisfies the initial conditions $\vec{\delta}(0)=\vec{0}$ and $\dot{\vec{\delta}}(0)=\vec{0}$ is

$$\delta_x(t)=\alpha\frac{\cos\omega_0 t}{\omega_0}\frac{y_{\rm osc}A^{-3/2}}{\omega_0K^2}\frac{\Delta-1}{\Delta}+\alpha\frac{\sin\omega_0 t}{\omega_0}\frac{y_{\rm osc}A^{-3/2}}{\omega_0}\left[\frac{\sin\omega_0 t\cos\omega_0 t}{\Delta}+\frac{F-E}{K^2}\right], \tag{16}$$

$$\delta_y(t)=\alpha\frac{\cos\omega_0 t}{\omega_0}\frac{bA^{-3/2}}{\omega_0K'^2}\frac{\cos\omega_0 t-\Delta}{\Delta}+\alpha\frac{\sin\omega_0 t}{\omega_0}\frac{bA^{-3/2}\sin\omega_0 t}{\omega_0\Delta}, \tag{17}$$

where we use $\omega_p=\omega_0$ and introduce the following variables in Eq. (16) and (17):

$$A=(b^2+y_{\rm osc}^2),\quad \Delta=\sqrt{1-K^2\sin^2\omega_0 t},$$
$$F=\int_0^{\omega_0 t}\frac{d(\omega_0 t)}{\sqrt{1-K^2\sin^2\omega_0 t}},$$
$$E=\int_0^{\omega_0 t}d(\omega_0 t)\sqrt{1-K^2\sin^2\omega_0 t},$$
$$K'^2=1-K^2. \tag{18}$$

In addition to the oscillatory electron motion in the laser field, electrons are assumed to have an average nonoscillatory drift velocity $V_0$. Once this velocity is obtained, electrons move away from their scattering centers in the time between two successive collisions. As a result, electron deflections do not average to zero and a net energy gain takes place. Corrections to the energy absorption that depend on the drift velocity $V_0$ will not be considered in this work and can be safely left out of consideration as long as many collisions occur within a laser period ($y_{\rm osc}\gg V_0 t$). As a result of the random-walk character of the electronic motion under the influence of the net long-range polarization electric field within a laser period, the quasineutrality ($Zn_{0i}\approx n_{0e}$) of the plasma will still be maintained.

## III. STOPPING POWER OF ELECTRONS

The total energy of the oscillator corresponding to the deviation $\vec{\delta}=\delta_x\hat{i}+\delta_y\hat{j}$ is as follows:

$$\epsilon_{\rm total}(b,t)=\frac{1}{2}m_e\dot{\vec{\delta}}^2+\frac{1}{2}m_e\omega_p^2\vec{\delta}^2. \tag{19}$$

Substitution of $\delta_x$ and $\delta_x$ from Eqs. (16) and (17) into Eq. (19) results in the following expression for the total energy:

$$\begin{aligned}\epsilon_{\rm total}(b,t)=&\frac{1}{2}m_e\alpha^2\frac{y_{\rm osc}^2}{\omega_0^2}\frac{A^{-3}}{K^4\Delta^2}+\frac{1}{2}m_e\alpha^2\frac{b^2}{\omega_0^2}\frac{A^{-3}}{K'^4\Delta^2}\cos^4\omega_0 t+\frac{1}{2}m_e\alpha^2\frac{y_{\rm osc}^2}{\omega_0^2}\frac{A^{-3}}{\Delta^2}\left[\sin\omega_0 t\cos\omega_0 t+\frac{\Delta(F-E)}{K^2}\right]^2\\&+\frac{1}{2}m_e\alpha^2\frac{b^2}{\omega_0^2}\frac{A^{-3}}{\Delta^2}\sin^4\omega_0 t+m_e\alpha^2\frac{b^2}{\omega_0^2}\frac{A^{-3}}{K'^2\Delta^2}\sin^2\omega_0 t\cos^2\omega_0 t+\frac{1}{2}m_e\alpha^2\frac{y_{\rm osc}^2}{\omega_0^2}\frac{A^{-3}}{\Delta^2}\sin^2\omega_0 t\cos^4\omega_0 t\\&+\frac{1}{2}m_e\alpha^2\frac{y_{\rm osc}^2}{\omega_0^2}\frac{A^{-3}}{\Delta^6}\sin^2\omega_0 t\cos^4\omega_0 t[1-2K^2\sin^2\omega_0 t+K^4\sin^4\omega_0 t]-m_e\alpha^2\frac{y_{\rm osc}^2}{\omega_0^2}\frac{A^{-3}}{\Delta^4}\sin^2\omega_0 t\\&\times\cos^4\omega_0 t[1-K^2\sin^2\omega_0 t]+m_e\alpha^2\frac{y_{\rm osc}^2}{\omega_0^2}\frac{A^{-3}}{K^2\Delta^2}\sin^2\omega_0 t\cos^2\omega_0 t-m_e\alpha^2\frac{y_{\rm osc}^2}{\omega_0^2}\frac{A^{-3}}{K^2\Delta^4}\sin^2\omega_0 t\cos^2\omega_0 t\end{aligned}$$

$$\times[1-K^2\sin^2\omega_0 t]+m_e\alpha^2\frac{y_{osc}^2}{\omega_0^2}\frac{A^{-3}}{\Delta^2}\sin^2\omega_0 t\cos^4\omega_0 t-m_e\alpha^2\frac{y_{osc}^2}{\omega_0^2}\frac{A^{-3}}{\Delta^4}\sin^2\omega_0 t\cos^4\omega_0 t[1-K^2\sin^2\omega_0 t]$$

$$+m_e\alpha^2\frac{y_{osc}^2}{\omega_0^2}\frac{A^{-3}}{K^2\Delta}\sin\omega_0 t\cos^3\omega_0 t(F-E)+m_e\alpha^2\frac{y_{osc}^2}{\omega_0^2}\frac{A^{-3}}{K^2\Delta^3}\sin\omega_0 t\cos^3\omega_0 t(F-E)$$

$$+2m_e\alpha^2\frac{b^2}{\omega_0^2}\frac{A^{-3}}{K'^4\Delta^2}\sin^2\omega_0 t\cos^2\omega_0 t[1-2K'^2+K'^4]+\frac{1}{2}m_e\alpha^2\frac{b^2}{\omega_0^2}\frac{K^4A^{-3}}{K'^4\Delta^6}\sin^2\omega_0 t\cos^2\omega_0 t[\cos^4\omega_0 t$$

$$+2K'^2\sin^2\omega_0 t\cos^2\omega_0 t+K'^4\sin^4\omega_0 t]+2m_e\alpha^2\frac{b^2}{\omega_0^2}\frac{K^2A^{-3}}{K'^4\Delta^4}\sin^2\omega_0 t\cos^2\omega_0 t$$

$$\times[K'^2\sin^2\omega_0 t+K'^2\cos 2\omega_0 t-\cos^4\omega_0 t], \tag{20}$$

where $K'$ for $y_{osc}\gg V_0 t$ can be written as

$$K'^2=1-K^2=1-\frac{y_{osc}^2}{b^2+y_{osc}^2}=\frac{b^2}{b^2+y_{osc}^2}.$$

Energy loss due to all possible collisions of an electron with ions that are uniformly distributed with density $n_{0i}$ between the collision parameters $b$ and $db+b$ can be evaluated as follows:

$$d\epsilon_{total}(b,t)=n_{0i}\epsilon_{total}(b,t)\sigma(\Omega)d\Omega dx$$

$$=n_{0i}\epsilon_{total}(b,t)bdbd\phi dx.$$

It is assumed that an electron passes through a distance much smaller than its mean free path during a period of laser oscillation, and that within one cycle, many collisions occur giving each $b$ equal probability.

The energy loss per unit length (stopping force) is

$$\frac{d\epsilon_{total}(b,t)}{dx}=n_{0i}\epsilon_{total}(b,t)bdbd\phi. \tag{21}$$

The average energy loss per unit length is obtained by integrating Eq. (21) over all directions $\phi\in[0,2\pi]$ and collision parameters $b$ as well as over one laser oscillation,

$$\left\langle\frac{d\epsilon_{total}(b,t)}{dx}\right\rangle_{b,\text{cycle}}=\frac{1}{2\pi}\int_0^{2\pi}d(\omega_0 t)\times\left[2\pi n_{0i}\int_0^{\infty}\epsilon_{total}(b,t)bdb\right],$$

where $\langle\rangle_{b,\text{cycle}}$ means averaging over all collision parameters $b$ and one laser oscillation. We introduce now two cutoff parameters $b_{min}$ and $b_{max}$ to avoid divergences for small and large $b$ values. The divergence for small $b$ values results from the use of an approximate relative separation according to Eq. (7). Using a self-consistent kinetic description of the electron-ion interactions in plasmas with finite temperatures will lead to a convergent behavior at large distances, since plasma polarization effects are correctly included in the formulation of the kinetic theory (plasma dielectric theory). $b_{min}$ is usually determined as follows [22,27,28]:

$$b_{min}=\max(b_{\perp},\lambda_B),\quad \lambda_B=\frac{\hbar}{m v_{rel}},\quad b_{\perp}=\frac{q_1q_2}{4\pi\epsilon_0 m_r v_{rel}^2}, \tag{22}$$

where $\lambda_B$ is the de Broglie wavelength and $v_{rel}$ is the electron-ion relative speed. In a strong laser field, the oscillation amplitude of an electron $y_{osc}=v_{osc}/\omega_0$ can be larger than the Debye length and therefore we cannot use $b_{max}=\lambda_D$ as a measure of the extent of the Coulomb interaction. Taking into account the presence of the laser field, a cutoff parameter $b=b_{max}$ can be introduced as follows:

$$b_{max}=\min\left(\frac{\langle v_{total}\rangle}{\omega_p},\frac{\langle v_{total}\rangle}{\omega_0}\right)=\frac{\langle v_{total}\rangle}{\omega_p}=\frac{\langle v_{total}\rangle}{\omega_0}, \tag{23}$$

where $v_{total}$ means the total time-averaged velocity given by

$$\frac{1}{2}mv_{total}^2=\frac{1}{2}m(v_{th}^2+v_{osc}^2\sin^2\omega_0 t), \tag{24}$$

$$\langle v_{total}\rangle_{\text{cycle}}=\sqrt{v_{th}^2+\frac{v_{osc}^2}{2}}\Leftrightarrow b_{max}=\sqrt{\lambda_D^2+\frac{y_{osc}^2}{2}}. \tag{25}$$

Integrating $\epsilon_{total}(b,t)$ in Eq. (20) over $b$ and upon substitution for $b_{max}$ from Eq. (25), we get after a lengthy calculation the following:

$$\left\langle\frac{d\epsilon_{total}(b,t)}{dx}\right\rangle_b\approx\frac{\pi}{2}\frac{n_{0i}m_e\alpha^2}{v_{osc}^2}\left[4.5+2\ln\frac{\sqrt{\lambda_D^2+\frac{1}{2}y_{osc}^2}}{b_{min}}+4\cos 2\omega_0 t-\frac{2}{3}\cos 4\omega_0 t+\frac{3}{2}\sin 2\omega_0 t+\frac{1}{2}\sin 4\omega_0 t+\frac{1}{4}\sin^3 2\omega_0 t\right]. \tag{26}$$

Equation (26) gives the instantaneous drag force $F_r$ resulting from the interaction of the ions with a single electron, which is equal to the net force that it will suffer.

## IV. COLLISION FREQUENCY

The electron-ion collision frequency $\nu_{\mathrm{ei}}$ for $v_{\mathrm{osc}} \gg v_{\mathrm{th}}$, can be defined as follows:

$$F_r = \nu_{\mathrm{ei}}(t) m_e v_{\mathrm{tot}}(t). \tag{27}$$

Substituting $F_r$ from Eq. (26) into Eq. (27) gives

$$\nu_{\mathrm{ei}}(t) = \frac{\pi}{2} \frac{n_{0i}\alpha^2}{v_{\mathrm{osc}}^2 \sqrt{v_{\mathrm{th}}^2 + v_{\mathrm{osc}}^2 \sin^2 \omega_0 t}} \Bigg[ 4.5 + 2 \ln \frac{\sqrt{\lambda_D^2 + \frac{1}{2} y_{\mathrm{osc}}^2}}{b_{\min}} + 4 \cos 2\omega_0 t - \frac{2}{3} \cos 4\omega_0 t + \frac{3}{2} \sin 2\omega_0 t + \frac{1}{2} \sin 4\omega_0 t + \frac{1}{4} \sin^3 2\omega_0 t \Bigg]. \tag{28}$$

To average the collision frequency in Eq. (28) over a laser oscillation in the limit $v_{\mathrm{osc}} \gg v_{\mathrm{th}} \approx 0$, we introduce the following expansion:

$$\frac{1}{\sqrt{v_{\mathrm{th}}^2 + v_{\mathrm{osc}}^2 \sin^2 \omega_0 t}} = \begin{cases} \dfrac{1}{v_{\mathrm{th}}}, & 0 \leq \omega_0 t \leq \omega_0 t_0 \\ \dfrac{1}{v_{\mathrm{osc}} \sin \omega_0 t}, & \omega_0 t_0 \leq \omega_0 t \leq \pi - \omega_0 t_0 \\ \dfrac{1}{v_{\mathrm{th}}}, & \pi - \omega_0 t_0 \leq \omega_0 t \leq \pi. \end{cases} \tag{29}$$

According to Eq. (29), $v_{\mathrm{osc}} \sin \omega_0 t$ is smaller than $v_{\mathrm{th}}$ in the two ranges $\omega_0 t \in [0, \omega_0 t_0]$ and $\omega_0 t \in [\pi - \omega_0 t_0, \pi]$. The value of $\omega_0 t_0$ that makes the condition $v_{\mathrm{osc}} \gg v_{\mathrm{th}}$ valid is obtained as follows:

$$\lim_{t \to t_0} \frac{1}{v_{\mathrm{th}}} = \lim_{t \to t_0} \frac{1}{v_{\mathrm{osc}} \sin \omega_0 t} \Leftrightarrow \omega_0 t_0 = \sin^{-1} \frac{v_{\mathrm{th}}}{v_{\mathrm{osc}}} \approx \frac{v_{\mathrm{th}}}{v_{\mathrm{osc}}}. \tag{30}$$

The time average of each term $g(\omega_0 t)$ in Eq. (28) can be evaluated using the following procedure:

$$\langle g(\omega_0 t) \rangle_{\mathrm{cycle}} = \frac{1}{2\pi} \int_0^{2\pi} g(\omega_0 t) d\omega_0 t = \lim_{t_0 \to 0} \frac{1}{\pi} \int_0^{\omega_0 t_0} g(\omega_0 t) d\omega_0 t + \lim_{t_0 \to 0} \frac{1}{\pi} \left[ \int_{\omega_0 t_0}^{\pi - \omega_0 t_0} g(\omega_0 t) d\omega_0 t + \int_{\pi - \omega_0 t_0}^{\pi} g(\omega_0 t) d\omega_0 t \right]. \tag{31}$$

The time-averaged collision frequency for $v_{\mathrm{osc}} \gg v_{\mathrm{th}}$ becomes

$$\langle \nu_{\mathrm{ei}} \rangle_{\mathrm{cycle}} = \frac{2 n_{0i}}{v_{\mathrm{osc}}^3} \left( \frac{Ze^2}{4\pi\epsilon_0 m_e} \right)^2 \left[ 1 + \ln \frac{2 v_{\mathrm{osc}}}{v_{\mathrm{th}}} \right] \ln \frac{\sqrt{\lambda_D^2 + \frac{1}{2} y_{\mathrm{osc}}^2}}{b_{\min}} + \frac{2 n_{0i}}{v_{\mathrm{osc}}^3} \left( \frac{Ze^2}{4\pi\epsilon_0} \right)^2 \left[ -1 + 4 \ln \frac{2 v_{\mathrm{osc}}}{v_{\mathrm{th}}} \right], \tag{32}$$

where we substitute for $\alpha$ from Eq. (7). The time-averaged energy absorption rate can be calculated as follows:

$$\langle \dot{\epsilon} \rangle_{\mathrm{cycle}} = \frac{1}{2} m_e n_{0e} \langle \nu_{\mathrm{ei}} \rangle_{\mathrm{cycle}} \left( \frac{1}{2} v_{\mathrm{osc}}^2 + v_{\mathrm{th}}^2 \right) = \frac{n_{0i} n_{0e}}{2 m_e v_{\mathrm{osc}}} \left( 1 + 2 \frac{v_{\mathrm{th}}^2}{v_{\mathrm{osc}}^2} \right) \left( \frac{Ze^2}{4\pi\epsilon_0} \right)^2 \left[ 1 + \ln \frac{2 v_{\mathrm{osc}}}{v_{\mathrm{th}}} \right] \times \ln \frac{\sqrt{\lambda_D^2 + \frac{1}{2} y_{\mathrm{osc}}^2}}{b_{\min}} + \frac{n_{0i} n_{0e}}{2 m_e v_{\mathrm{osc}}} \left( 1 + 2 \frac{v_{\mathrm{th}}^2}{v_{\mathrm{osc}}^2} \right) \times \left( \frac{Ze^2}{4\pi\epsilon_0} \right)^2 \left[ -1 + 4 \ln \frac{2 v_{\mathrm{osc}}}{v_{\mathrm{th}}} \right]. \tag{33}$$

## V. RESULTS AND DISCUSSION

The basic equation of the oscillator model [Eq. (4)] describes the dynamic of free electrons that are harmonically bound to the ion scattering centers. Accounting for spatial variation and temperature dependence of the electron deflection, the model equation takes the following general form:

$$\frac{\partial^2 \vec{\delta}}{\partial t^2} - v_{\mathrm{th}}^2 \vec{\nabla}^2 \vec{\delta} + \omega_{\mathrm{pe}}^2 \vec{\delta} = - \frac{Ze^2}{4\pi\epsilon_0 m_e} \frac{\vec{r}_e(t)}{|\vec{r}_e(t)|^3}. \tag{34}$$

We notice that Eq. (4) does not include the spatial variations of the electron deflection and therefore does not account for excitation of waves in plasma or for mechanisms that are responsible for transferring energy from waves to particles. This assumption is valid for the initial stage of laser-matter interaction. In the initial stage of plasma formation and heating, electron-ion collision frequency can be as large as $10^{16}\ \mathrm{s}^{-1}$ and therefore collisional absorption is appreciable [20].

Ionic motion can be ignored in the short time scale of laser-matter interaction. In this case, collective absorption mechanisms involving ion sound waves and the two-plasmon decay do not take place. At oblique incidence, resonance absorption may take place. For a nearly normal incident light, absorption due to the electron-ion collisions may play a principal role [29,30].

Starting from the ballistic dynamical equation [Eq. (4)], known as the oscillator model, we solve the model equation in the presence of an intense laser field ($v_{\mathrm{osc}} \gg v_{\mathrm{th}}$) at the plasma critical density ($\omega_0 = \omega_p$). Time-resolved and time-averaged electron stopping force and electron-ion collision frequency are determined [Eq. (26), (28), and (32)]. An expression for the time-averaged energy absorption rate is given [Eq. (33)].

Following the literature on the interaction of an intense laser field with matter, we find only asymptotic expressions for the electron-ion collision frequency and the energy absorption rate which are valid for a combination of the asymptotic limits $\omega_0 \gg \omega_p$, $\omega_0 \ll \omega_p$, ($v_{\rm osc} \gg v_{\rm th}$), and ($v_{\rm osc} \ll v_{\rm th}$). No attention has been paid to the transition regions ($\omega_0 \approx \omega_p$) and ($v_{\rm osc} \approx v_{\rm th}$). Inclusion of transition points through extrapolation of the asymptotic results from the left and right sides can be right or wrong, and needs a physical justification [22].

To our knowledge, there are no previous works concerning the time-resolved electron stopping force and electron-ion collision frequency and the time-averaged electron stopping force and electron-ion collision frequency at $\omega_0 \approx \omega_p$. Comparison of our results with previous works is not possible and is restricted to the structure of the derived expressions. Similarities are found in the dependence of the collision frequency on $v_{\rm osc}^{-3}$ and of the energy absorption rate on $v_{\rm osc}^{-1}$ for intense laser fields . Characteristic to all linear calculations (no electron capture) is the $Z^2$ dependence.

It is important to note that the second term on the right-hand side of Eq. (33) is independent of the cutoff parameters $b_{\rm min}$ and $b_{\rm max}$, which are characteristics of single-particle interactions. This term accounts for energy absorption due to pure collective effects that are responsible for the generation of the laser harmonics. The first term on the right-hand side of Eq. (33) consists of two parts that are dependent on $\ln(b_{\rm max}/b_{\rm min})$ and $\ln(b_{\rm max}/b_{\rm min})\ln(2v_{\rm osc}/v_{\rm th})$. The part proportional to $\ln(b_{\rm max}/b_{\rm min})$ only accounts for pure single-particle interactions, and the other is due to the interaction between electrons and the generated laser harmonics.


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